

# System Modelling and Design

## COMP2111



Johannes Åman Pohjola

+ tutors:

Zhuo (Zoey) Chen

Raphael Douglas Giles

Formal

# System Modelling and Design

## COMP2111



Johannes Åman Pohjola

+ tutors:  
Zhuo (Zoey) Chen  
Raphael Douglas Giles

Formal

# System Modelling and Design COMP2111

Credit for the material  
also goes to:

Paul Hunter,  
Christine Rizkallah,  
Liam O'Connor,  
and Carroll Morgan



Johannes Åman Pohjola

+ tutors:  
Zhuo (Zoey) Chen  
Raphael Douglas Giles

# We'll learn to

model systems in a way that's unambiguous and mathematically precise.

# We'll be able to

say what it means for a system to satisfy its specification,  
and prove that it does so.

# We'll need

a substantial toolbox of discrete math and formal logic.

Don't worry; we'll teach it, not assume it.

# Non-examples

September 1981

Transmission Control Protocol  
Functional Specification

## 3.9. Event Processing

The processing depicted in this section is an example of one possible implementation. Other implementations may have slightly different processing sequences, but they should differ from those in this section only in detail, not in substance.

The activity of the TCP can be characterized as responding to events. The events that occur can be cast into three categories: user calls, arriving segments, and timeouts. This section describes the processing the TCP does in response to each of the events. In many cases the processing required depends on the state of the connection.

Events that occur:

User Calls

OPEN  
SEND  
RECEIVE  
CLOSE  
ABORT  
STATUS

Arriving Segments

SEGMENT ARRIVES

Timeouts

USER TIMEOUT  
RETRANSMISSION TIMEOUT  
TIME-WAIT TIMEOUT

The model of the TCP/user interface is that user commands receive an immediate return and possibly a delayed response via an event or pseudo interrupt. In the following descriptions, the term "signal" means cause a delayed response.

Error responses are given as character strings. For example, user commands referencing connections that do not exist receive "error: connection not open".

Please note in the following that all arithmetic on sequence numbers, acknowledgment numbers, windows, et cetera, is modulo  $2^{32}$  the size of the sequence number space. Also note that " $=<$ " means less than or equal to (modulo  $2^{32}$ ).

# Non-examples

This RFC is a specification in English.

Natural language specs tend to have:

- Ambiguities
- Room for interpretation
- Important details in the writer's head absent from actual text.

September 1981

Transmission Control Protocol  
Functional Specification

## 3.9. Event Processing

The processing depicted in this section is an example of one possible implementation. Other implementations may have slightly different processing sequences, but they should differ from those in this section only in detail, not in substance.

The activity of the TCP can be characterized as responding to events. The events that occur can be cast into three categories: user calls, arriving segments, and timeouts. This section describes the processing the TCP does in response to each of the events. In many cases the processing required depends on the state of the connection.

Events that occur:

### User Calls

OPEN  
SEND  
RECEIVE  
CLOSE  
ABORT  
STATUS

### Arriving Segments

SEGMENT ARRIVES

### Timeouts

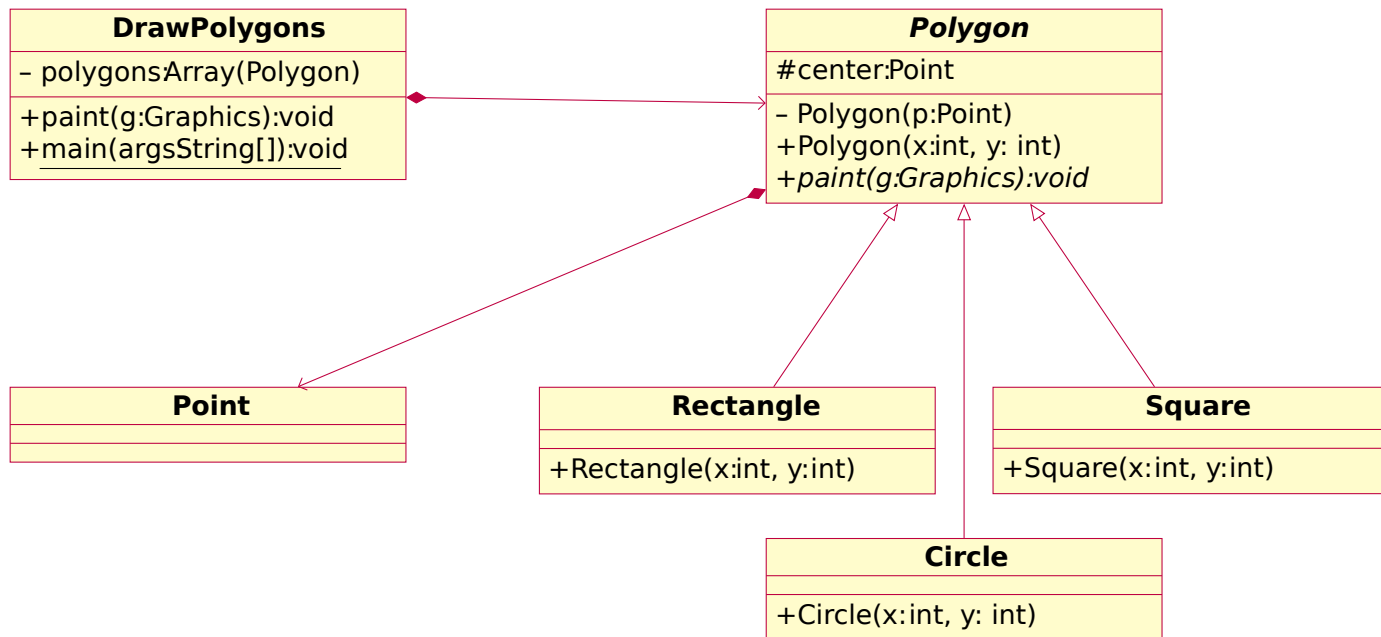
USER TIMEOUT  
RETRANSMISSION TIMEOUT  
TIME-WAIT TIMEOUT

The model of the TCP/user interface is that user commands receive an immediate return and possibly a delayed response via an event or pseudo interrupt. In the following descriptions, the term "signal" means cause a delayed response.

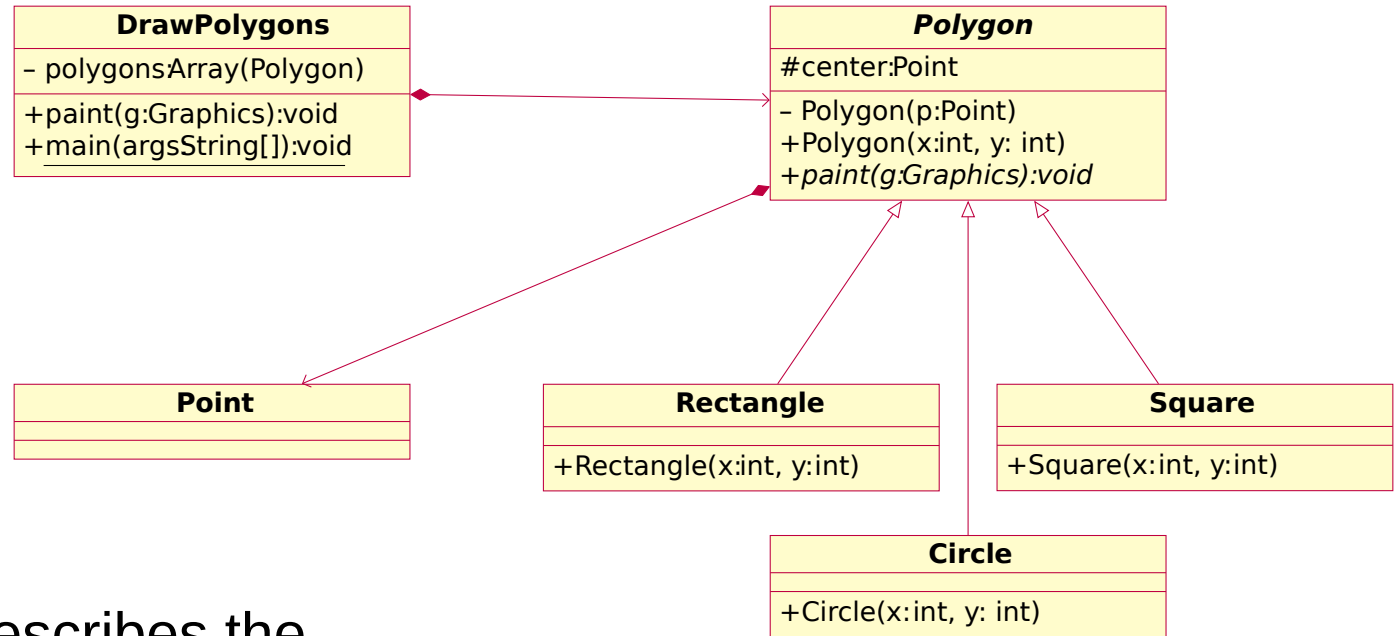
Error responses are given as character strings. For example, user commands referencing connections that do not exist receive "error: connection not open".

Please note in the following that all arithmetic on sequence numbers, acknowledgment numbers, windows, et cetera, is modulo  $2^{32}$  the size of the sequence number space. Also note that " $=<$ " means less than or equal to (modulo  $2^{32}$ ).

# Non-examples



# Non-examples



This UML diagram describes the *structure* of the system, not its behaviour.



# Resources

Course website: <http://www.cse.unsw.edu.au/~cs2111>

- Lecture slides, tutorials
- Assignment instructions
- ...

Ed forum: <https://edstem.org/au/courses/15105/>

- General announcements
- Class discussion, announcements
- E-mail [cs2111@cse.unsw.edu.au](mailto:cs2111@cse.unsw.edu.au) if you haven't been invited!

Moodle: <https://moodle.telt.unsw.edu.au/>

- Lecture recordings
- Weekly quizzes

# Examination

- Weekly quizzes: 15 credits total
  - After the lectures of every week (except W6 and W10).
  - Will appear on Moodle.
  - Deadline: Monday 4PM (before start of next week's lectures)
- Three assignments (individual or pair, written):  $11+12+12=35$  credits
- Final exam (online, format TBA): 50 credits

# Introduction to “Formal” Logic

Start here

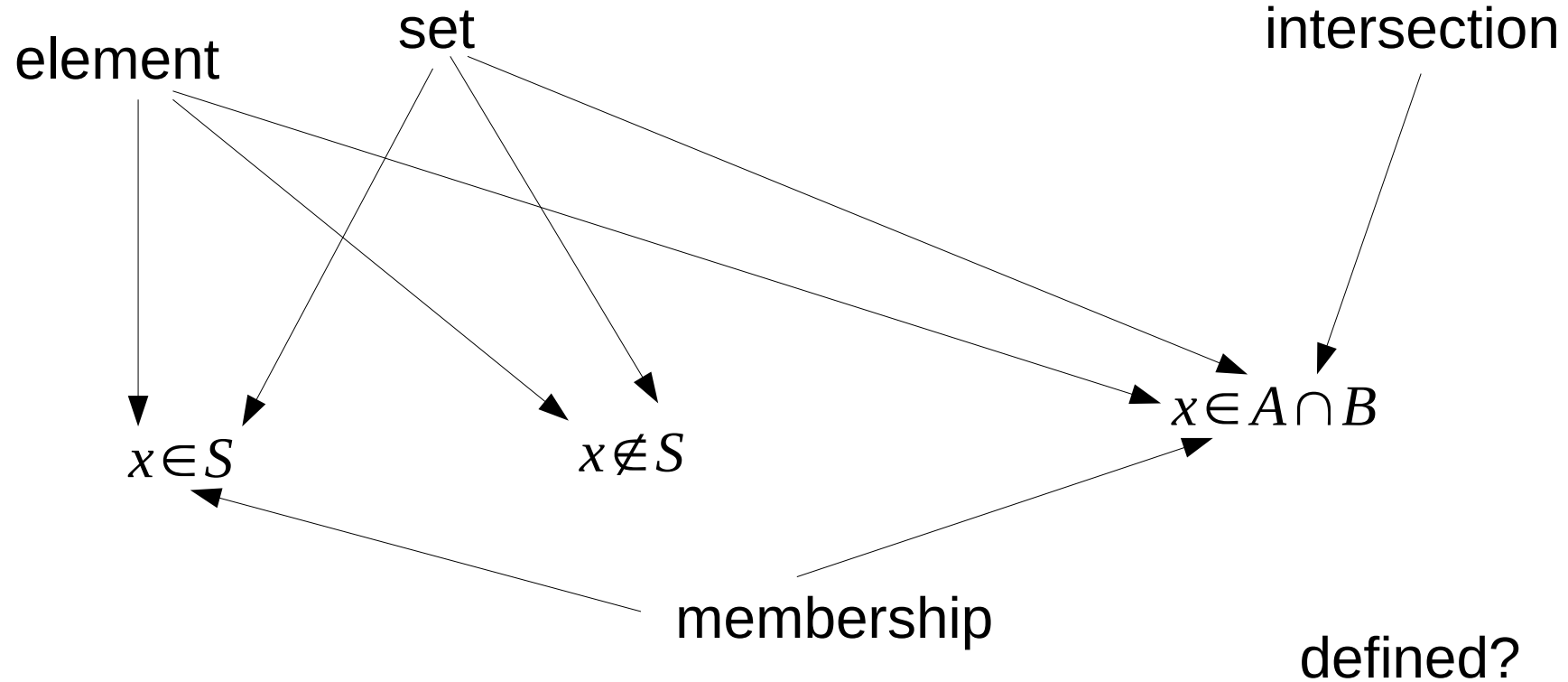


This is a (draft) textbook for  
COMP6721 (In-)Formal  
Methods by Carroll Morgan

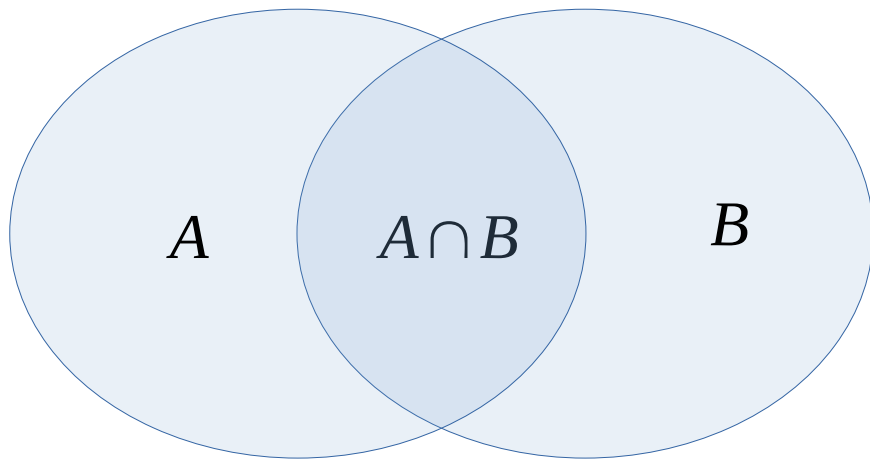
It's on the course website.

<b>D</b>	<b>The arithmetic of conditions</b>	<b>207</b>
D.1	Introduction and rationale . . . . .	207
D.2	Why is my program correct? . . . . .	207
D.3	How do I write my program in the first place? . . . . .	209
D.4	Calculating with conditions . . . . .	210
D.5	Simple calculations in logic . . . . .	212
D.6	Terms . . . . .	213
D.7	Simple formulae . . . . .	214
D.8	Propositions, and propositional formulae . . . . .	215
D.9	Operator precedence . . . . .	215
D.10	Calculation with logical formulae . . . . .	217
D.11	<i>Exercises</i> on propositions . . . . .	218
D.12	Quantifiers . . . . .	221
D.13	<i>Exercises</i> on quantifiers . . . . .	222
D.14	(General) formulae . . . . .	223
<b>E</b>	<b>Some helpful logical identities</b>	<b>225</b>
E.1	Some basic propositional rules . . . . .	225
E.2	Some basic predicate rules . . . . .	228
E.3	<i>Exercises</i> on rules for logic . . . . .	232
E.4	Epilogue on notation and terminology . . . . .	232

# Introduction to “Formal” Logic



# Introduction to “Formal” Logic



$x \in A \cap B$

if and only if

$x \in A$

and  $x \in B$

# Introduction to “Formal” Logic

intersection  $A \cap B$

union  $A \cup B$

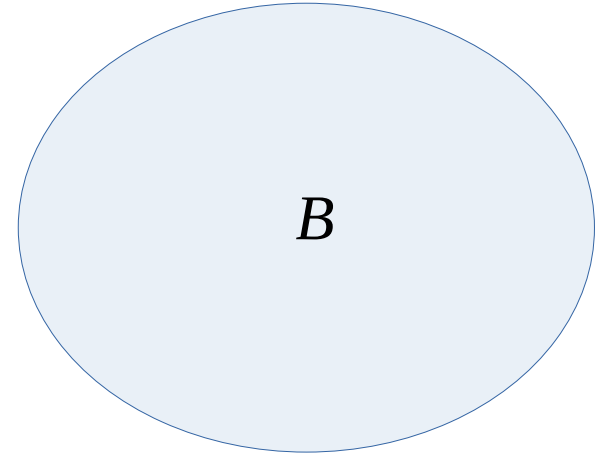
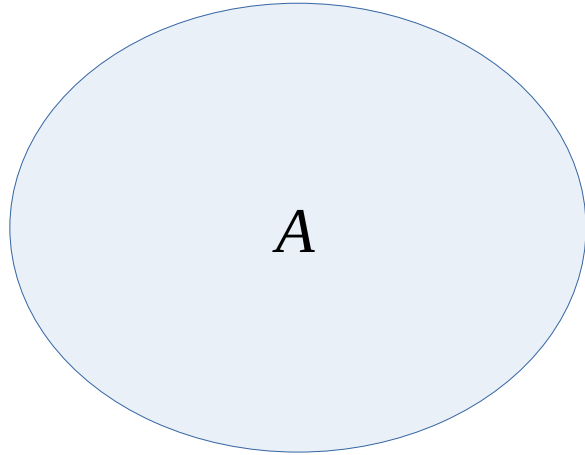
subset  $A \subseteq B$

Q: Subset is not like others  
in an important way. How?

$x \in A \cup B$  if and only if  $x \in A$  or  $x \in B$

$A \subseteq B$  if and only if  $x \in A$  then  $x \in B$

# Introduction to “Formal” Logic



Where is  $A \cap B$  now?

# Introduction to “Formal” Logic

Let's prove  $A \subseteq A \cup B$



# Why so pedantic?

$\{y \mid y \subseteq x\}$  The set of all subsets of  $x$   
aka the powerset of  $x$

$\{y \mid y \in x\}$  The set of all elements of  $x$   
  
Aka just  $x$

# Why so pedantic?

$x \in x$        $x$  is an element of  $x$   
(The set that contains itself)  
Does it make sense to write?

Is it ever true?

$\{ x \mid x \in x \}$       The set of all sets that contain themselves

Aka the empty set

# Why so pedantic?

$$\{ x \mid x \notin x \}$$

$$y = \{ x \mid x \notin x \}$$

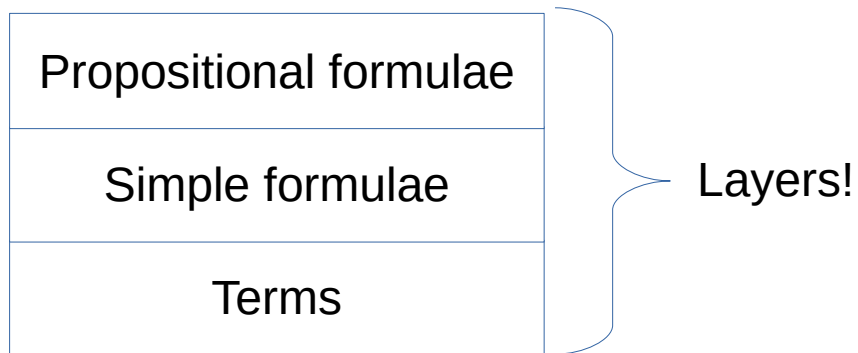
$$y \in y \Rightarrow y \notin y$$

$$y \notin y \Rightarrow y \in y$$

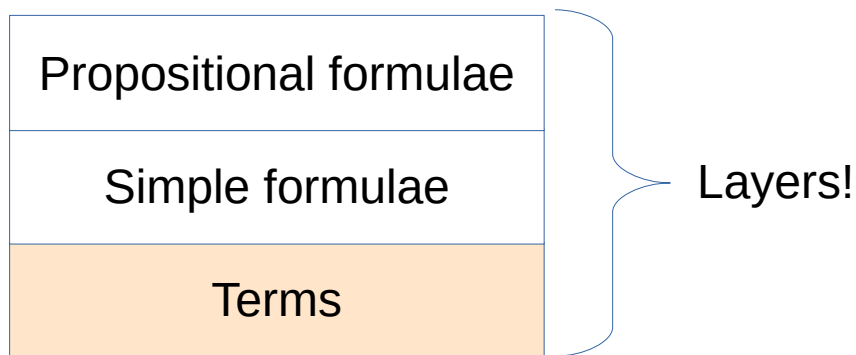
“There is just one point where I  
have encountered a difficulty”  
- Bertrand Russell

Q: Why does this matter?

# The language of logic



# The language of logic

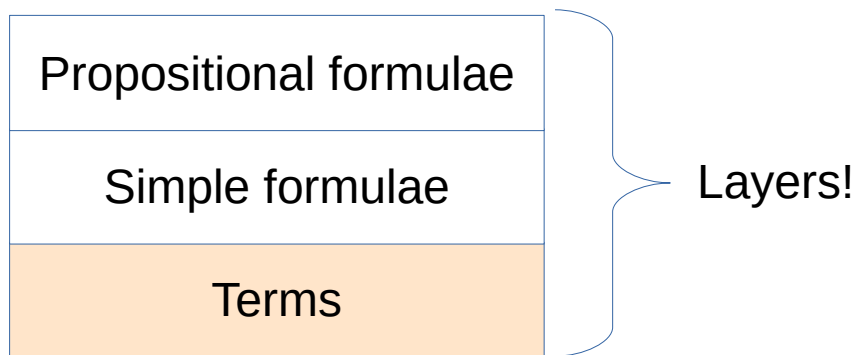


A *term* is either

- (a) a variable, or
- (b) a constant symbol, or
- (c) a function symbol applied to the correct number of other terms.

A function's number of arguments is its *arity*.

# The language of logic



A *term* is either  
(a) a variable, or  
(b) a constant symbol, or  
(c) a function symbol applied to the correct number of other terms.

A function's number of arguments is its *arity*.

variables

$x, y, \dots$

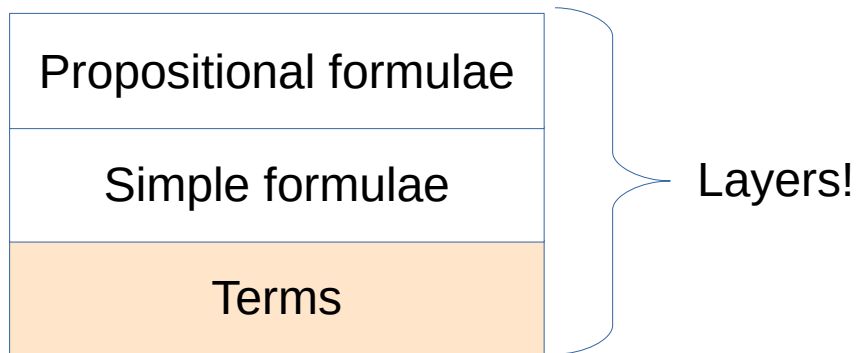
Constants

$1, \{\}, \pi$

functions

union, intersection  
 $+, -, !$

# The language of logic



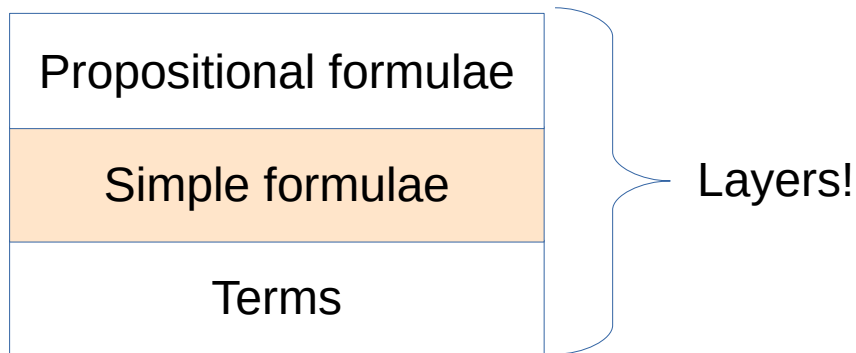
$x+1$   
 $x \quad 0$   
 $\sin(x/2)$

Terms have *values*

$x \ y$   
 $x+$   
 $x++$

Not terms

# The language of logic



A *simple formula* is a predicate symbol applied to the correct number of (term) arguments.

$t < u$

$t = u$

$t \geq u$

$even(t)$

$t \in u$

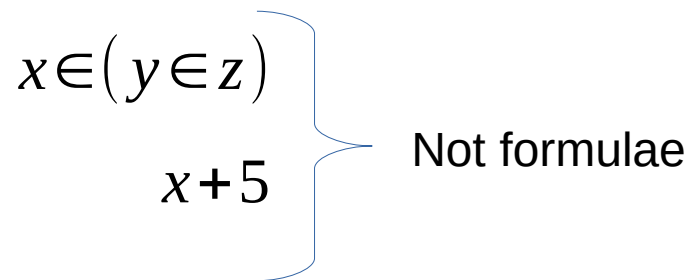
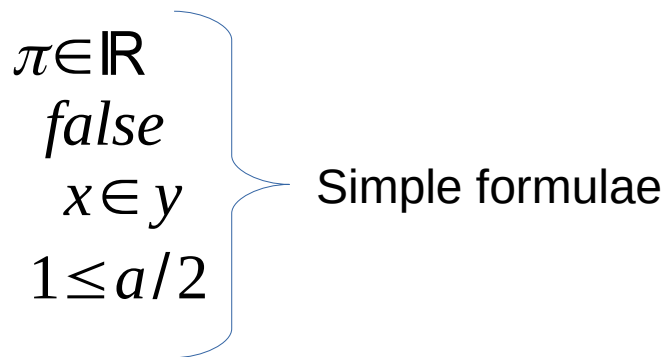
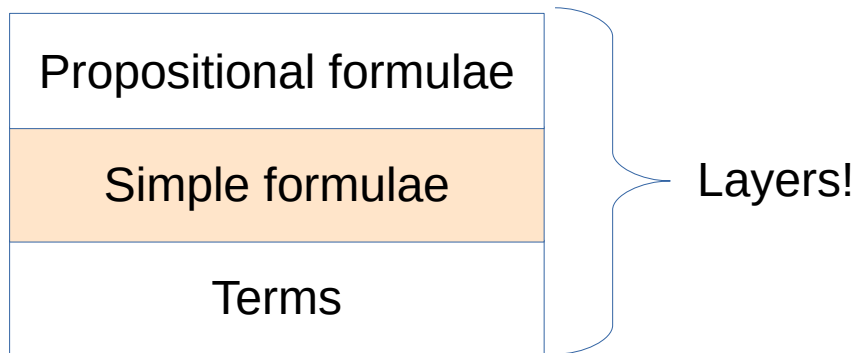
$false$

$t, u$  are terms

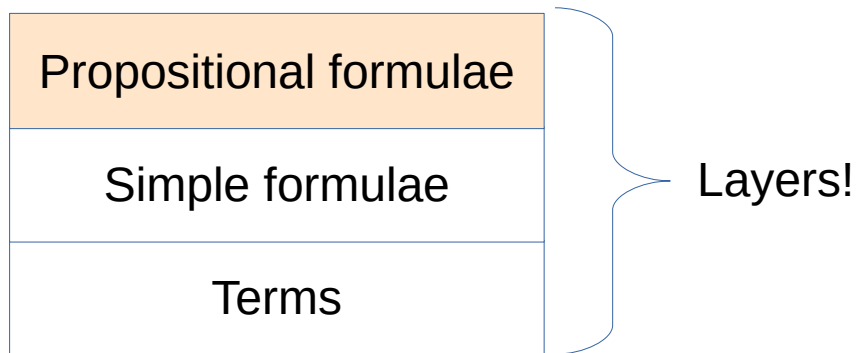
These can be True  
or False



# The language of logic



# The language of logic

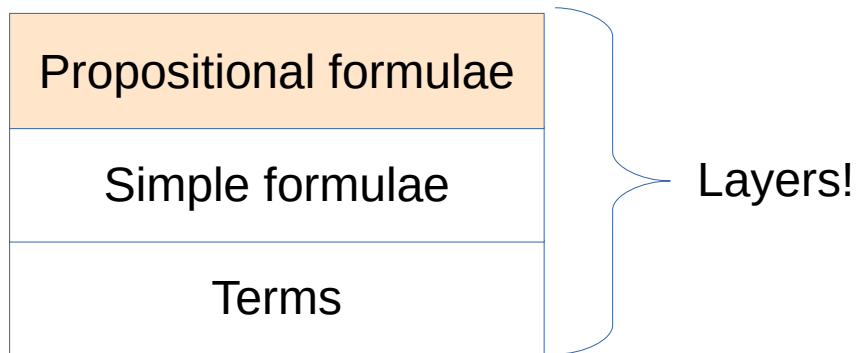


A *propositional formula* is either  
(a) a simple formula  
(b) a *propositional connective* applied to  
the right number of arguments.



- $\wedge$  Conjunction (and)
- $\vee$  Disjunction (or)
- $\neg$  negation
- $\rightarrow$  implication
- $\leftrightarrow$  biimplication

# The language of logic



*false*

$x \in y \wedge x \geq 2$

$n \neq n \leftrightarrow (n = 1 \vee n = 3)$

Q: Is this formula True?

# Truth tables

A	B	$A \wedge B$
True	True	True
True	False	False
False	False	False
False	True	False

A	B	$A \rightarrow B$
True	True	True
True	False	False
False	False	True
False	True	True

A	B	$A \vee B$
True	True	True
True	False	True
False	False	False
False	True	True

A	B	$A \leftrightarrow B$
True	True	True
True	False	False
False	False	True
False	True	False

# The language of logic: summary

Propositional formulae	Propositional connectives
Simple formulae	Predicate symbols
Terms	Constants, functions, variables

Truth tables define what the propositional connectives mean.

Q: Did the layered, systematic approach help against Russell's paradox?

# Calculating with logic

Now we're here

<b>D</b>	<b>The arithmetic of conditions</b>	<b>207</b>
D.1	Introduction and rationale . . . . .	207
D.2	Why is my program correct? . . . . .	207
D.3	How do I write my program in the first place? . . . . .	209
D.4	Calculating with conditions . . . . .	210
D.5	Simple calculations in logic . . . . .	212
D.6	Terms . . . . .	213
D.7	Simple formulae . . . . .	214
D.8	Propositions, and propositional formulae . . . . .	215
D.9	Operator precedence . . . . .	215
D.10	Calculation with logical formulae . . . . .	217
D.11	<i>Exercises</i> on propositions . . . . .	218
D.12	Quantifiers . . . . .	221
D.13	<i>Exercises</i> on quantifiers . . . . .	222
D.14	(General) formulae . . . . .	223
<b>E</b>	<b>Some helpful logical identities</b>	<b>225</b>
E.1	Some basic propositional rules . . . . .	225
E.2	Some basic predicate rules . . . . .	228
E.3	<i>Exercises</i> on rules for logic . . . . .	232
E.4	Epilogue on notation and terminology . . . . .	232

# Calculating with logic

Propositional formulae

Are like the conditions in if-then-else, while

Terms

are like the RHS of assignment statements

We can *calculate* with logic as a thinking tool for programming,

...just as we can use mathematical calculation as a thinking tool for physics.

# Calculating with logic

```
if l <= m < h:  
    ...  
else:  
    ... #what's true here?
```



Python syntax.

(let's calculate)



# Calculating with logic

```
if l <= m < h:  
    thing1  
elif m < l:  
    thing2  
else:  
    thing3
```

```
if l <= m < h:  
    thing1  
elif m >= h:  
    thing3  
else:  
    thing2
```

Are these programs the same?

(let's calculate)